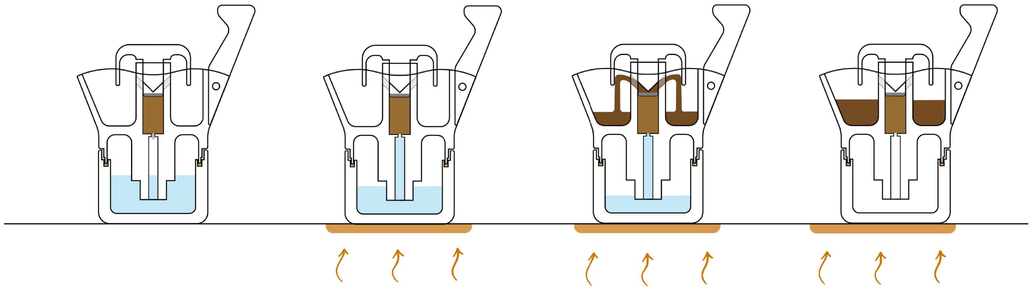


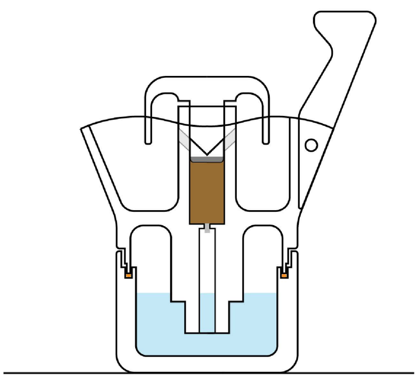
DESIGN PARAMETERS

Desired pressure for filtration: 4-5 bar

Final temperature of coffee: 368 K

Volume of coffee produced: (2 shots) ~70-80 mL

The ideal dimensions for the puck of coffee grounds must be found to match the ideal-condition parameters. The desired pressure was based off of feasible pressure produced by heating of the bottom reservoir. The aim was to increase the pressure from the 1-2 bar that a standard "moka pot" style coffee percolator achieves to 4-5 bar, closer to the values used to brew "authentic" espresso.

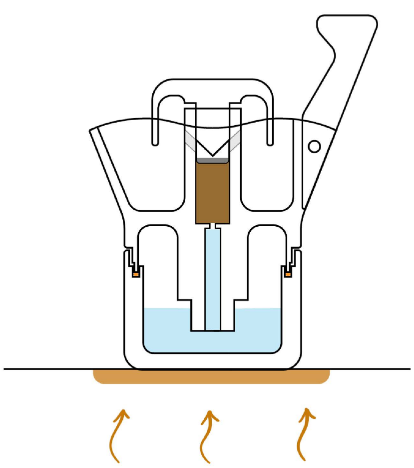


EQUATING INTERNAL PRESSURE TO THE PRESSURE NEEDED TO PERCOLATE COFFEE:

To achieve a more accurate value for the pressure, the internal pressure of the bottom reservoir was matched to the pressure needed to push liquid up the central channel + the pressure needed for the liquid to filter through the coffee grounds see Equation (1).

$$(1)\quad P_t = P_f + P_h$$

where P_t is the total pressure inside the reservoir, P_f is the filtration pressure necessary, P_h is the pressure due to the height difference.



CALCULATION OF INTERNAL PRESSURE (from air pressure and vapor pressure) INSIDE THE BOTTOM RESERVOIR WHILE HEATING:

The internal pressure of the reservoir has contributions from attempted expansion of the air pockets trapped above the water and the vapor pressure caused by the heating of the water (see Equation (2)). Both of these factors are dependent on the temperature (T). The final temperature was set to 95 degrees Celsius later in the calculations to match ideal values for coffee brewing.

$$(2)\quad P_t = P_{air}(T) + P_{sat}(T)$$

where $P_{air}(T)$ is the pressure from expansion of the air due to increased temperature. $P_{sat}(T)$ is the saturated vapor pressure.

PRESSURE DUE TO HEATING OF TRAPPED AIR:

For the pressure from the attempted expansion of air, I used a modified ideal gas equation (Equation (3)) as a reasonable approximation. The initial and final volumes are roughly equal as we are just looking to find the pressure when filtration begins. Overtime, the water level will decrease, and thus this partial pressure will decrease (as the final volume of the air grows).

$$(3)\quad P_{air}(T) = \frac{P_{air}(T_i) \cdot V_i \cdot T}{T_i \cdot V}$$

where $P_{air}(T_i)$ is the initial pressure, T_i and V_i are the intial temperature and pressure of the air, T and V are the temperature and volume of the air at a given moment.

PRESSURE DUE TO WATER VAPOR FORMATION:

Similarly we find the partial pressure from the saturated water vapor at a given temperature using Equation (4). This equation is derived from the Clausius Clapeyron equation which assumes that the vapor will perform as an ideal gas. This is a somewhat inaccurate simplification at high temperatures. However, within the range we are concerned by, it should give a reasonable estimation.

$$(4)\quad P_{sat}(T) = P_{sat}(T_i) * \exp\left(\frac{L \cdot M}{R}\left(\frac{1}{T_i} - \frac{1}{T}\right)\right)$$

where $P_{sat}(T_i)$ is the initial saturated vapor pressure, L is the heat of vaporization, M is the molecular weight of water, R is the gas constant, T_i and T are initial and current water temperature.

COMBINE FOR TOTAL INTERNAL PRESSURE:

Combining Equations (3) and (4) gives us the estimated pressure inside the bottom reservoir. Using dimensions from the Solidworks model and given values for the saturated vapor pressure of water, and the variables L, M, and R we find the total pressure is 4.3 bar, nicely within the desired range.

Internal pressure when filtration should begin: 4.3 bar.

PRESSURE AGAINST GRAVITY:

With this internal pressure to work from I solved for the other half of Equation (1) which combines the filtration pressure and the pressure needed to overcome the difference in elevation. Using an idealized Bernoulli Equation I found the expected pressure just to overcome the height difference (see Equation (5)). This ended up being less than 1 kPa (substantially less than the pressures achieved within the bottom reservoir).

$$(5)\quad \text{delta } P_{\text{due to height}} = \rho * g * h2 - \rho * g * h1$$

where ρ is the density of water, g is the gravitational constant, h2 is the height at the top of the percolator, h1 is the bottom of the percolator.

FILTRATION PRESSURE:

This means that the bulk of the right hand side of Equation (1) is due to the pressure required to push the fluid through the filtration puck of coffee grounds. The equation for this pressure was estimated with Darcy's Law for linear filtration (see Equation (6)). This equation requires the filtration constant (k) for coffee grounds, I used a value derived by Concetto Gianino (see references at the bottom of the page) for a "moka" style coffee pot. This value will change based on whether the coffee grounds are packed in, the fineness of the grind etc. however, having a value to work from will keep the design inside a functional range.

$$(6)\quad P_f = \frac{m * h * u}{k * S * \rho * t}$$

where P_f is the pressure of filtration, m is mass, h is the height of filtration, u is the fluid viscocity, k is the filtration constant for coffee grounds, S is the surface area of filtration, rho is the density of fluid, t is the time for filtration.

SOLVE FOR UNKNOWN DIMENSIONS:

Thus, equating the internal pressure of 4.3 bar to the pressure needed to overcome height gain and the pressure required for filtration it was possible to solve for h, the height of the coffee ground puck. This turned out to be 1.25" for a surface area of pi/4 square inches (the filtration area).

TAKEAWAYS

This analysis resulted in the **dimensions of coffee grounds area (height = 1.5", diameter = 1")** and the theoretical maximum pressure inside the bottom reservoir to be **4.3 bar**.